

## INTEGRATED SUPPLY CHAIN COST MANAGEMENT MATHEMATICAL MODEL : A TWO- ECHELON APPROACH

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### Abstract:

This paper is presenting an integrated supply chain cost management model in which a two-echelon approach has been considered. In this model we are focusing on optimizing costs and improving coordination between the vendor and supplier. Key cost components, including holding costs, production costs, deterioration costs, rework costs, and setup costs, are comprehensively analyzed for both echelons. The model considers sequential flow of goods, functional specialization, and interdependence between the vendor and supplier. By leveraging this approach, organizations can achieve improved efficiency, reduced costs, and enhanced performance within their supply chain operations. The findings underscore the importance of integration and collaboration in achieving supply chain excellence, offering valuable insights for practitioners and researchers alike.

*Keywords: Supply chain, inventory, deterioration, two- echelon*

### Introduction:

In today's complex and dynamic business environment, effective supply chain management is critical for organizations seeking to maintain competitiveness and meet customer demands. Central to supply chain optimization is the coordination and integration of various stages or echelons within the network. This integration ensures a smooth flow of goods, efficient resource utilization, and cost-effective operations.

Supply chains are comprised of multiple echelons, each representing a distinct stage in the production and distribution process. These echelons range from suppliers and manufacturers to distributors, retailers, and ultimately the end consumers. The management of inventory across these echelons is a critical aspect of supply chain efficiency. Each echelon holds inventory for different purposes, incurs different types of costs, and faces distinct operational challenges. Optimizing inventory management across these echelons is essential for minimizing costs, maximizing efficiency, and meeting customer service level requirements.

The bullwhip effect, a well-documented phenomenon in supply chain management, highlights the importance of coordination and integration across echelons. This effect describes the amplification of demand variability as orders move up the supply chain from retailers to wholesalers to manufacturers. Lee et al. (1997) explain that this effect can lead to excessive inventory levels, poor customer service, and increased operational costs. Addressing the bullwhip effect requires a deep understanding of the interactions between different echelons and the implementation of integrated inventory management strategies.

In their comprehensive guide on supply chain design and management, Simchi-Levi et al. (2008) emphasize the importance of integrating supply chain activities to achieve overall efficiency. They argue that successful supply chain management involves not only the coordination of logistics and operations but also the alignment of incentives and information sharing across all echelons. This holistic approach ensures that all parts of the supply chain work towards common goals, reducing inefficiencies and improving performance.

Chopra and Meindl (2019) further elaborate on the role of inventory management in supply

chain success. They discuss various inventory strategies and their impact on supply chain performance, highlighting the need for balancing holding costs, ordering costs, and stockout risks. By considering these factors, organizations can develop inventory policies that optimize the total cost of ownership across different echelons.

The integration of supply chain finance is another critical aspect of achieving cost efficiency and coordination. Sodhi and Tang (2012) explore the interplay between supply chain operations and financial considerations, arguing that integrated supply chain finance can help mitigate risks, reduce costs, and improve overall supply chain resilience. They suggest that by aligning financial strategies with operational objectives, organizations can create more robust and responsive supply chains.

This paper explores the concept of integrated supply chain cost management through the lens of a two-echelon inventory model. Specifically, we focus on the interactions and processes between the vendor (manufacturer) and the supplier (distributor) tiers within the supply chain. By analyzing key cost components such as holding costs, production costs, deterioration costs, rework costs, and setup costs for both echelons, we aim to develop insights into how organizations can achieve cost efficiency and coordination within their supply chain operations.

### **Literature Review:**

The management of inventory within supply chains has been a critical area of research for decades, with numerous studies focusing on optimizing costs and improving efficiency across various echelons. This review highlights key contributions to the field, particularly those related to integrated supply chain models, inventory cost management, and the coordination of multi-echelon systems.

#### ***Integrated Supply Chain Models***

Integrated supply chain models emphasize the importance of coordination and collaboration among different stages of the supply chain to achieve overall efficiency and cost reduction. Simchi-Levi et al. (2008) argue that successful supply chain management involves not only the coordination of logistics and operations but also the alignment of incentives and information sharing across all echelons. This approach ensures that all parts of the supply chain work towards common goals, thereby reducing inefficiencies and improving performance. Chopra and Meindl (2019) further elaborate on the role of inventory management in achieving supply chain success. They discuss various inventory strategies and their impact on supply chain performance, highlighting the need to balance holding costs, ordering costs, and stockout risks. By considering these factors, organizations can develop inventory policies that optimize the total cost of ownership across different echelons.

Recent studies continue to emphasize the importance of integrated approaches. For instance, Ivanov and Dolgui (2020) discuss the resilience of supply chains in the face of disruptions, underscoring the need for integrated risk management strategies across all echelons to ensure continuity and efficiency. Similarly, Queiroz et al. (2021) highlight the role of digital technologies in enhancing supply chain integration, suggesting that technologies like blockchain and IoT can facilitate better coordination and information sharing.

#### ***Multi-Echelon Inventory Management***

The concept of multi-echelon inventory management addresses the challenges of managing inventory across various stages of the supply chain. Lee et al. (1997) introduced the bullwhip effect, which describes how small fluctuations in consumer demand can cause progressively larger fluctuations in orders and inventory levels as one moves up the supply chain. This phenomenon underscores the importance of coordination between echelons to minimize excessive inventory levels, poor customer service, and increased operational costs. Graves and Willems (2000) present a strategic inventory placement model that aims to determine optimal inventory levels across multiple echelons. Their model helps organizations identify the most cost-effective locations for holding inventory, taking into account factors such as demand variability, lead times, and service level requirements. This approach provides a framework for improving overall supply chain efficiency by strategically positioning inventory where it can have the most significant impact.

More recent research by Raj, Jayaraman, and Jain (2020) examines the implementation of multi-echelon inventory systems in the context of sustainability. Their findings suggest that integrating

sustainability practices within multi-echelon systems can lead to significant cost savings and environmental benefits, further highlighting the importance of a coordinated approach.

### ***Cost Optimization in Supply Chains***

Effective cost management is a critical component of supply chain optimization. The various cost elements associated with inventory management, such as holding costs, setup costs, production costs, deterioration costs, and rework costs, need to be carefully analyzed and managed. Zipkin (2000) provides a comprehensive overview of inventory theory and the different cost components involved in inventory management. His work highlights the importance of balancing these costs to achieve overall cost efficiency in supply chains.

Sodhi and Tang (2012) explore the interplay between supply chain operations and financial considerations, arguing that integrated supply chain finance can help mitigate risks, reduce costs, and improve overall supply chain resilience. They suggest that by aligning financial strategies with operational objectives, organizations can create more robust and responsive supply chains. This integration of financial and operational strategies is critical for achieving cost efficiency and improving supply chain performance. More recent studies by Silva et al. (2021) delve into the role of advanced analytics in cost optimization, demonstrating how data-driven decision-making can enhance inventory management and reduce costs. Their research underscores the potential of leveraging big data and machine learning techniques to optimize supply chain operations.

### ***Rework and Deterioration Costs***

In addition to traditional inventory costs, rework and deterioration costs play a significant role in supply chain cost management. Nahmias (1982) discusses the impact of product deterioration on inventory policies, emphasizing the need for models that account for deterioration rates in perishable goods. His work provides valuable insights into how organizations can develop inventory strategies that minimize the costs associated with product deterioration. Additionally, rework costs, which arise from the need to repair or reprocess defective items, are a critical consideration in supply chain management. Chen and Ryan (2001) examine the cost implications of rework in manufacturing systems, highlighting the importance of incorporating rework costs into inventory models to accurately reflect the total cost of production.

## **MATERIALS AND METHODS:**

### **Assumptions and Notation:**

- 1 We are considering single type item for this inventory system.
- 2 Demand is  $D = \alpha - \beta S$
- 3 Shortages are not allowed for this paper.
- 4 Here in the paper we are considering one vendor and one supplier i.e. two echelon supply chain.
- 5 Demand rate for vendor and supplier both are same.

$I_v$ - vendor inventory

$I_s$ - supplier inventory

$P$ - production rate (vendor)

$D$ - demand rate (both vendor and supplier)

$\phi_v$ - Deterioration rate(vendor)

$\phi_s$ - Deterioration rate(supplier)

$T_1$ - Vendor production time

$T$ - shipment production time in case no production by vendor

$H_{cv}$ - holding cost (vendor)

$H_{cs}$ - holding cost (supplier)

$R_v$ - rework cost (vendor)

$S_v$ - setup cost(vendor)

$S_s$ - setup cost (supplier)

$d_v$ - deterioration cost (vendor)

$d_s$ - deterioration cost (supplier)

$\alpha$ - demand parameter

$\beta$ - demand parameter

a-production parameter

### Mathematical Modelling for Vendor

Let us discuss the inventory system for vendor. The time between  $[0, T_1]$  production of items happen and inventory increases and after  $T_1$  the inventory decreases due to combined effect of demand and deterioration.

#### Differential equations associated with vendors inventory

$$\begin{aligned} dI_{v1}/dt &= P-D- \phi_v I_{v1} \\ &= a(\alpha-\beta S)-(\alpha-\beta S)-\phi_v I_{v1}, \quad [0, T_1] \end{aligned} \quad (1)$$

With boundary condition  $I_{v1}(0) = 0$  ,  $I_{v1}(T_1) = MI_v$

$$\begin{aligned} dI_{v2}/dt &= -D- \phi_v I_{v2} \\ &= -(\alpha-\beta S)-\phi_v I_{v2}, \quad [T_1, T] \end{aligned} \quad (2)$$

With boundary condition  $I_{v2}(T) = 0$  ,

Solutions for the associated differential equations

$$I_{v1} = \frac{(\alpha-\beta S)}{\phi_v} (a-1) (1- e^{-\phi_v t}), \quad [0, T_1] \quad (1a)$$

$$I_{v2} = \frac{(\alpha-\beta S)}{\phi_v} (e^{\phi_v(T-t)} - 1) , \quad [T_1, T] \quad (2a)$$

#### Total cost associated with vendor

(a) **Holding cost:** A firm also has unsold inventory and the cost associated with the storing of these unsold inventory is called holding cost for the inventory. Total holding cost is:

$$\begin{aligned} H.C.V &= H_{cv} \left[ \int_0^{T_1} I_{v1}(t) dt + \int_{T_1}^T I_{v2}(t) dt \right] \\ &= H_{cv} \frac{(\alpha-\beta S)}{\phi_v} (a-1) \left[ T_1 + \frac{1}{\phi_v} (e^{-\phi_v T_1} - 1) \right] + H_{cv} \frac{(\alpha-\beta S)}{\phi_v} \left[ \frac{1}{\phi_v} (e^{\phi_v(T-T_1)} - 1) + T_1 - T \right] \end{aligned} \quad (3)$$

(b) **Production cost for the vendor:** Self manufacturing system is the system in which items are produced by the machine.

$$P.C .V = \omega + \frac{g}{P} + sP + \frac{(\alpha-\beta S)}{\phi_v} (a-1) (1- e^{-\phi_v T_1}), \quad (4)$$

$\omega$  is the material cost and it is fixed

(c) **Deterioration cost:**

$$D.C.V = (d_v)(\phi_v) \left\{ \frac{(\alpha-\beta S)}{\phi_v} (a-1) \left[ T_1 + \frac{1}{\phi_v} (e^{-\phi_v T_1} - 1) \right] + \frac{(\alpha-\beta S)}{\phi_v} \left[ \frac{1}{\phi_v} (e^{\phi_v(T-T_1)} - 1) + T_1 - T \right] \right\} \quad (5)$$

(d) **Rework cost:** Many times products are returned back. And returned product parts are reused or the fault in the returned products is corrected to resell it.

So, all the cost in this reverse manufacturing or which all included in reverse logistics is called remanufactured cost.

Let  $N = \left\{ \int_x^{T_1} P dt \quad \text{when } x < T_1 \right.$

$\left. \text{when } x \geq T_1 \right\}$

So expected defected items during production time

$E(N) = \int w P(T_1 - x) f(x) dx$  ( integration goes from 0 to  $T_1$ )

Where  $f(x)$  is the probability density function  $f(x) = \beta e^{-\beta x}$

Therefore

Rework cost

$$\begin{aligned} R.C.V &= R_v \int w P(T_1 - x) f(x) dx \\ &= R_v w P \delta (T_1^2/2) \end{aligned} \quad (6)$$

(e) **Set up cost:** This cost includes the cost in scheduling, moving the starting material, keeping the records, setting the production equipment and also testing few output units to be assured that the equipment is properly set up.

$$S.U.C.V = S_v \quad (7)$$

Total cost for vendor is the sum of all above costs:

$$T.C.V = \frac{1}{T} [H.C.V + P.C.V + R.C.V + S.U.C.V + D.C.V]$$

$$\frac{1}{T} \left[ \begin{aligned} &Hcv \frac{(\alpha - \beta S)}{\phi_v} (a - 1) [T_1 + \frac{1}{\phi_v} (e^{-\phi_v T_1} - 1)] + Hcv \frac{(\alpha - \beta S)}{\phi_v} \left[ \frac{1}{\phi_v} (e^{\phi_v (T - T_1)} - 1) + T_1 - T \right] \\ &\quad + \omega + \frac{g}{P} + sP + \frac{(\alpha - \beta S)}{\phi_v} (a - 1) (1 - e^{-\phi_v T_1}) \\ &+ (dv)(\phi_v) \left\{ \frac{(\alpha - \beta S)}{\phi_v} (a - 1) [T_1 + \frac{1}{\phi_v} (e^{-\phi_v T_1} - 1)] + \frac{(\alpha - \beta S)}{\phi_v} \left[ \frac{1}{\phi_v} (e^{\phi_v (T - T_1)} - 1) + T_1 - T \right] \right\} \\ &\quad + RvwP\delta \left( \frac{T_1^2}{2} \right) + S_v \end{aligned} \right]$$

### Mathematical modelling for supplier

During time  $[0, T_2]$  inventory of supplier decreases due to combined effect of demand and deterioration. Differential equations associated with suppliers inventory

$$\begin{aligned} dI_s/dt &= -D - \phi_s I_s \\ &= -(\alpha - \beta S) - \phi_s I_s, \end{aligned} \quad [0, T_2] \quad (8)$$

With boundary condition  $I_s(T_2) = 0$ ,

Solution for above differential equation

$$I_s = \frac{(\alpha - \beta S)}{\phi_s} (e^{\phi_s (T_2 - t)} - 1) \quad (9)$$

### Total cost associated with supplier

(a) **Holding cost-** A firm also has unsold inventory and the cost associated with the storing of these unsold inventory is called holding cost for the inventory.

$$\begin{aligned} H.C.S &= H_{cs} \left[ \int_0^{T_2} I_s(t) dt \right] \\ &= H_{cs} \frac{(\alpha - \beta S)}{\phi_s} \left[ \frac{1}{\phi_s} (e^{\phi_s T_2} - 1) - T_2 \right] \end{aligned} \quad (10)$$

(b) **Set up cost-** This cost includes the cost in scheduling, moving the starting material, keeping the records, setting the production equipment and also testing few output units to be assured that the equipment is properly set up.

$$S.U.C.S = S_s \quad (11)$$

(c) **Deterioration cost –**

$$\begin{aligned} D.C.S &= \phi_s d_s \left[ \int_0^{T_2} I_s(t) dt \right] \\ &= \phi_s d_s \frac{(\alpha - \beta S)}{\phi_s} \left[ \frac{1}{\phi_s} (e^{\phi_s T_2} - 1) - T_2 \right] \end{aligned} \quad (12)$$

Total cost for supplier

$$T.C.S = \frac{n}{T} [H.C.S + D.C.S + S.U.C.S]$$

$$= \frac{n}{T} \left[ Hcs \frac{(\alpha - \beta S)}{\phi_s} \left[ \frac{1}{\phi_s} (e^{\phi_s T_2} - 1) - T_2 \right] + \phi_s ds \frac{(\alpha - \beta S)}{\phi_s} \left[ \frac{1}{\phi_s} (e^{\phi_s T_2} - 1) - T_2 \right] \right]$$

Total cost of the inventory is

**T** = T.C.V + T.C.S

$$\left[ Hcv \frac{(\alpha - \beta S)}{\phi_v} (a - 1) \left[ T_1 + \frac{1}{\phi_v} (e^{-\phi_v T_1} - 1) \right] + Hcv \frac{(\alpha - \beta S)}{\phi_v} \left[ \frac{1}{\phi_v} (e^{\phi_v (T - T_1)} - 1) + T_1 - T \right] + \omega + \frac{g}{P} + sP + \frac{(\alpha - \beta S)}{\phi_v} (a - 1) (1 - e^{-\phi_v T_1}) + (dv)(\phi_v) \left\{ \frac{(\alpha - \beta S)}{\phi_v} (a - 1) \left[ T_1 + \frac{1}{\phi_v} (e^{-\phi_v T_1} - 1) \right] + \frac{(\alpha - \beta S)}{\phi_v} \left[ \frac{1}{\phi_v} (e^{\phi_v (T - T_1)} - 1) + T_1 - T \right] \right\} + RvwP\delta \left( \frac{T_1^2}{2} \right) + Sv \right]$$

$$+ \frac{n}{T} \left[ Hcs \frac{(\alpha - \beta S)}{\phi_s} \left[ \frac{1}{\phi_s} (e^{\phi_s T_2} - 1) - T_2 \right] + \phi_s ds \frac{(\alpha - \beta S)}{\phi_s} \left[ \frac{1}{\phi_s} (e^{\phi_s T_2} - 1) - T_2 \right] \right]$$

**RESULTS AND DISCUSSION:**

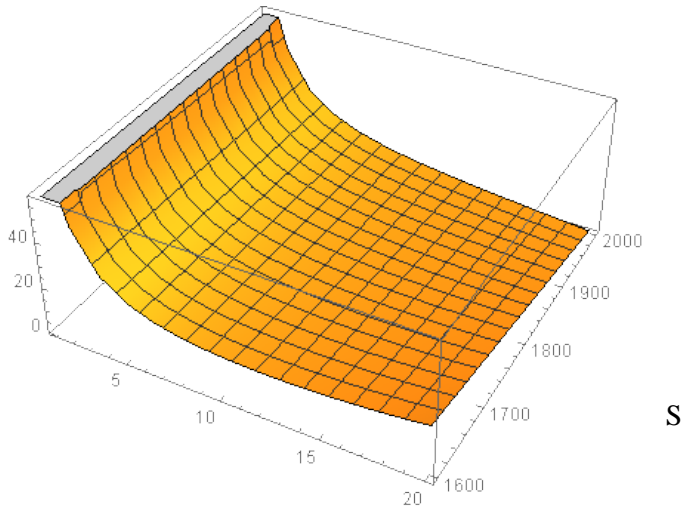
*Numerical analysis*

Parameter	value	Parameter	Value
$H_{cv}$	0.02	$\alpha$	190
$\beta$	0.1	$\phi_v, \phi_s$	0.1
$\omega$	0.01	$g$	0.9
$P$	0.01	$d_v$	0.03
$R_v$	0.1	$w$	50
$\delta$	90	$S_v$	0.2
$n$	5	$H_{cs}$	0.2
$s$	1	$d_s$	1100
$S_s$	0.1	$a$	1000

*Solution*

Parameter	Values
$T$	12.7175
$S$	1664.1
<i>Total cost</i>	14.4182

*Sensitivity analysis*



T<sub>2</sub>

Parameter	%	<i>T</i>	<i>S</i>	Total cost
<i>H<sub>cv</sub></i>	+20%	5.40569	264.785	31.5892
	+10%	5.18425	0.747408	32.1474
	-10%	5.72295	100.643	29.1489
	-20%	5.81954	0.34432	28.38
$\alpha$	+20%	5.37317	311.179	32.0427
	+10%	5.27261	98.0859	31.8109
	-10%	6.29177	360.699	27.0912
	-20%	6.2716	190.872	26.6389
$\beta$	+20%	6.65956	619.998	26.2742
	-10%	12.4253	1835.49	14.349
	-20%	6.56416	770.808	27.1259
$\varphi_v$	+20%	5.42829	201.043	30.6779
	+10%	5.43819	140.567	30.5955
	-10%	5.8978	258.457	28.957
	-20%	6.81179	735.563	25.0689
$\omega$	+20%	12.2592	1658.58	14.0636
	+10%	12.6982	1663.1	14.7313
	-10%	12.6904	1662.66	14.456
	-20%	12.7874	1667.8	14.318
<i>g</i>	+20%	7.16132	646.994	29.3109
	+10%	6.32142	425.225	29.7568
	-10%	5.31711	132.266	28.5344
	-20%	12.7175	1664.1	14.4182
<i>P</i>	+20%	6.07389	721.61	23.9953
	+10%	5.3006	105.92	28.8521
	-10%	7.1743	725.456	27.3712
	-20%	7.02081	702.88	29.5947
<i>d<sub>v</sub></i>	+20%	5.53739	155.291	30.4054
	+10%	5.41158	58.3025	30.8028
	-10%	7.60125	1011.67	23.2396
<i>R<sub>v</sub></i>	+20%	9.28535	1658.2	15.1375
	+10%	9.28535	1658.2	15.1375

Parameter	%	$T$	$S$	Total cost
$w$	-10%	12.2944	1658.56	14.0622
	-20%	12.3505	1659	14.0501
	+20%	6.2716	190.872	26.6389
	+10%	12.1346	1658.53	14.0731
	-10%	9.28535	1658.2	15.1375
$\delta$	-20%	9.28535	1658.2	15.1375
	+20%	12.984	1663.09	14.4132
	+10%	12.7207	1664.28	14.4132
	-10%	12.6556	1660.86	14.7902
	-20%	12.563	1667.45	14.946
$S_v$	+20%	6.46408	606.585	26.9722
	+10%	9.28535	1658.2	15.1397
	-10%	10.76	1658.53	14.0731
	-20%	45.36	1658.2	15.1375
	$n$	+20%	93.034	1658.2
+10%		23.856	1663.09	15.1375
-10%		11.498	1664.28	14.0622
-20%		48.934	1708.46	14.0501
$H_{cs}$		+20%	26.764	1763.94
	+10%	12.4286	1659.48	14.0366
	-10%	12.6556	1664.28	26.9722
	-20%	12.563	1660.86	15.1397
	$s$	+20%	5.45703	68.8574
+10%		5.4805	85.0487	30.4675
-10%		6.4535	80.943	31.47
-20%		5.5355	30.46	30.53
$d_s$		+20%	3.567	45.98
	+10%	5.43773	55.9541	30.6351
	-10%	6.093	59.0345	40.865
	-20%	7.0432	73.85	44.682
	$S_s$	+20%	9.80456	1634.98
+10%		9.2835	1658.2	15.1377
-10%		9.03215	1679.04	16.234
-20%		8.09743	1700.1	16.943
$a$		+20%	5.6345	255.98
	+10%	5.78422	266.933	29.3589
	-10%	5.69874	270.98	29.509
	-20%	5.68732	300.09	30.0987

**Result:****1. Parameter  $\beta$ :**

- The values of  $T$  (between 6.56416 and 12.4253) and  $S$  (between 619.998 and 1835.49) show significant variability.
- The total cost decreases significantly with increases in  $\beta$  (e.g., from 27.1259 at -20% to 14.349 at -10%).

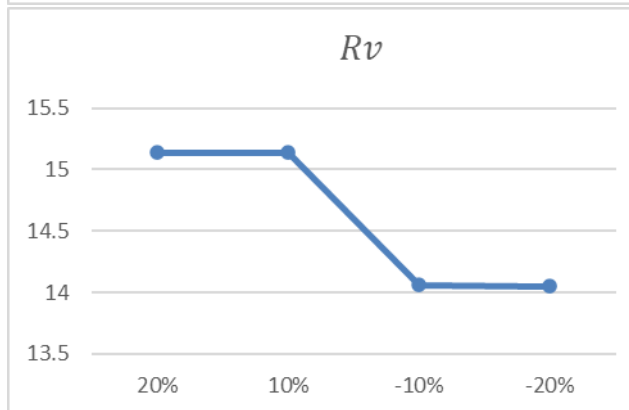
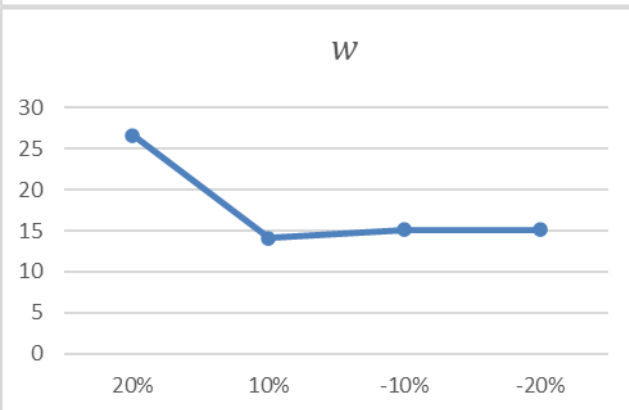
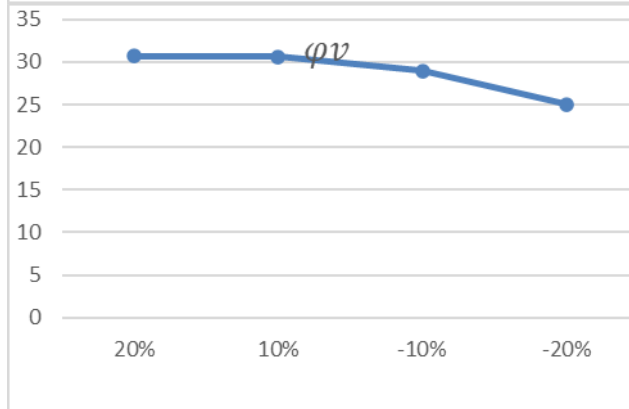
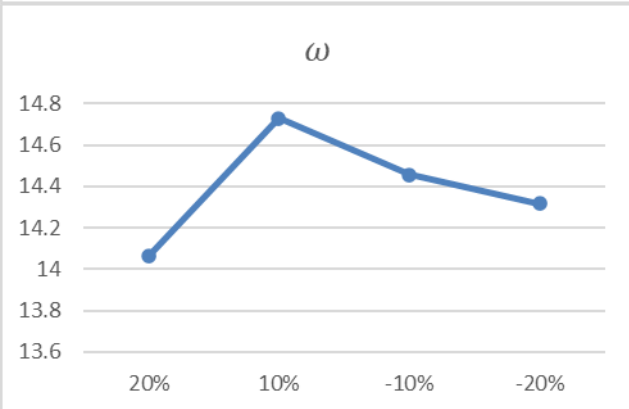
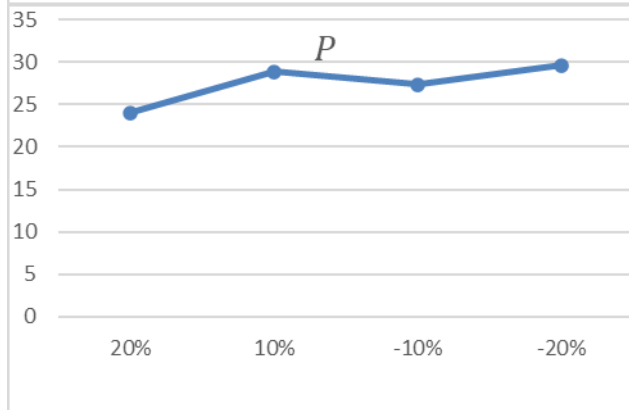
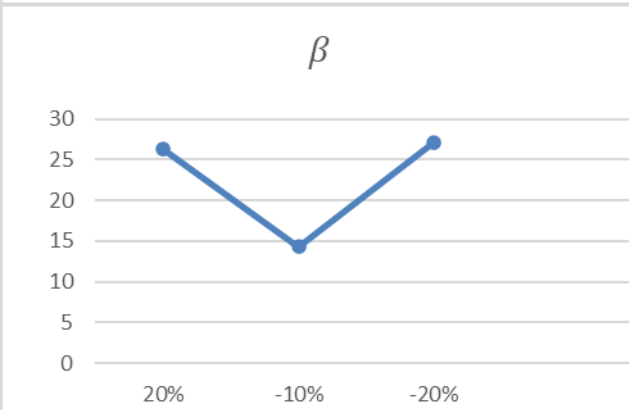
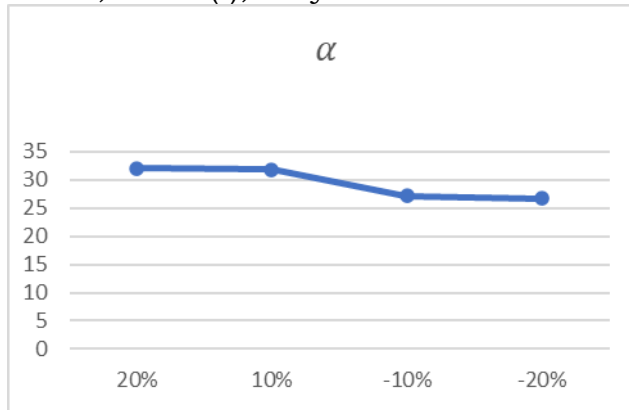
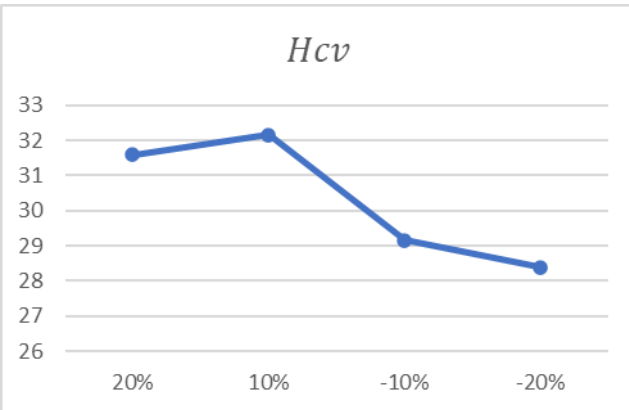
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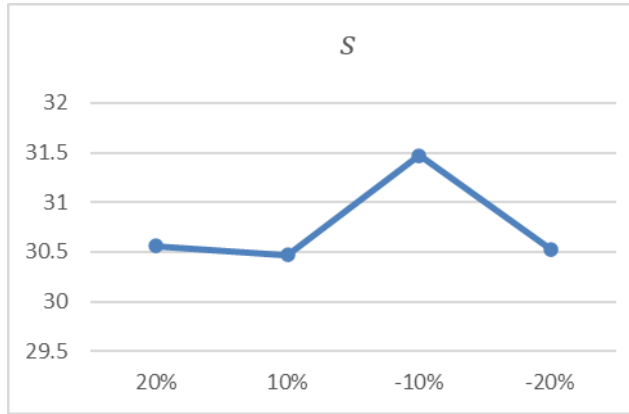
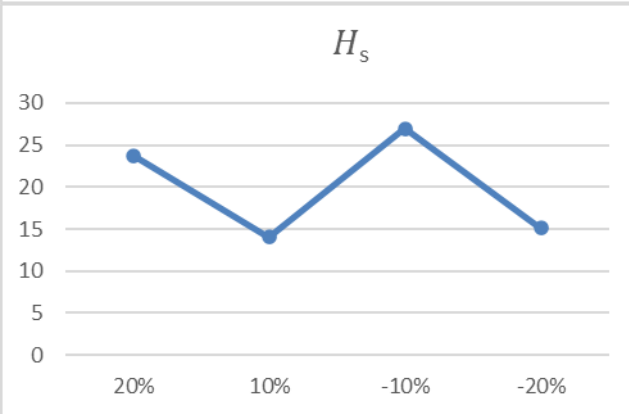
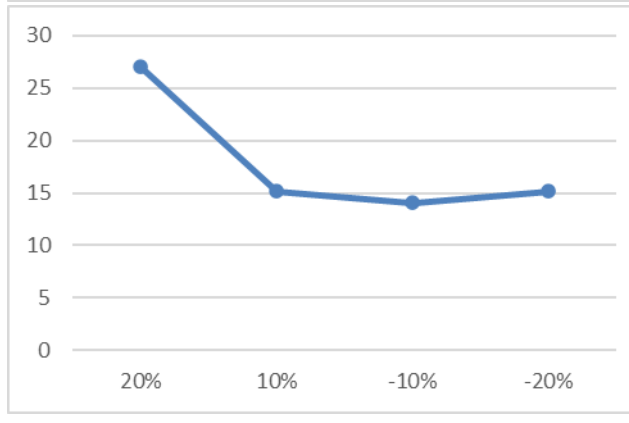
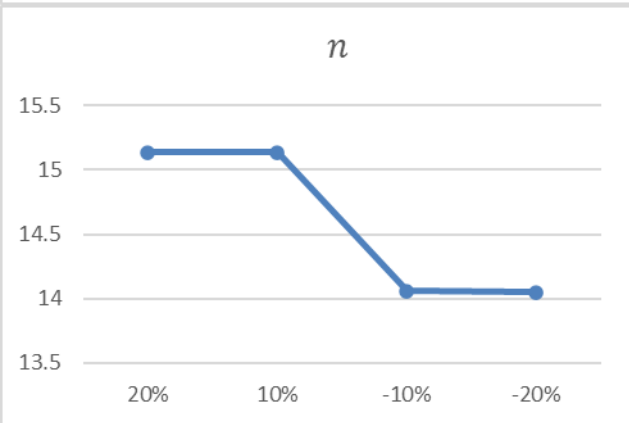
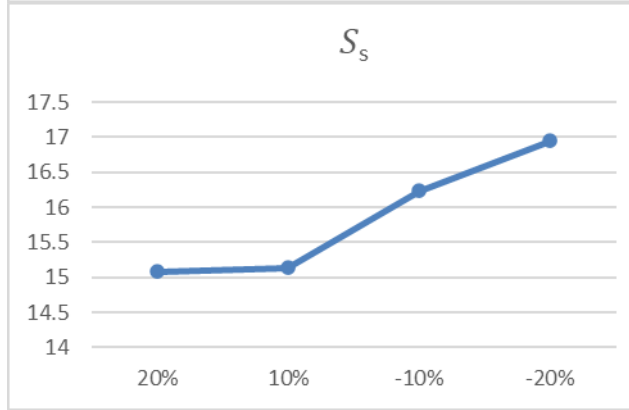
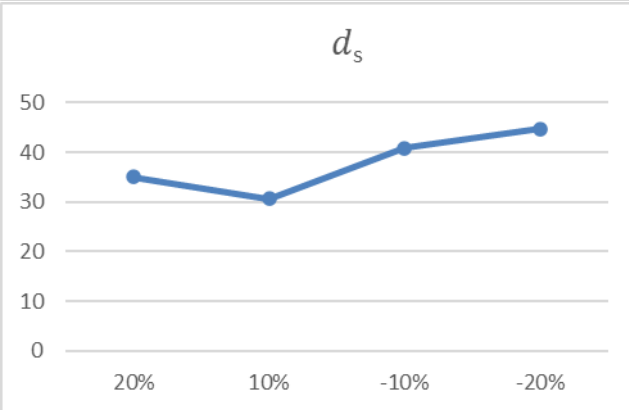
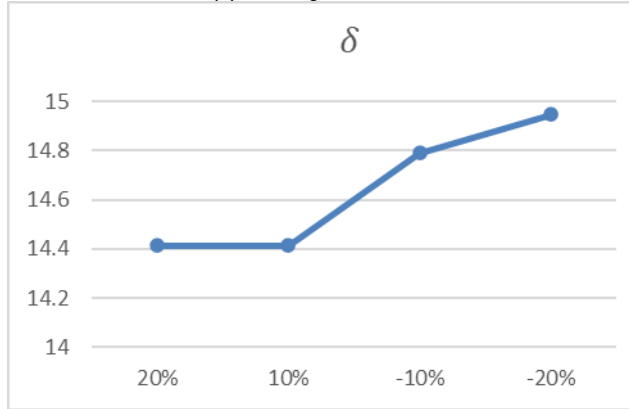
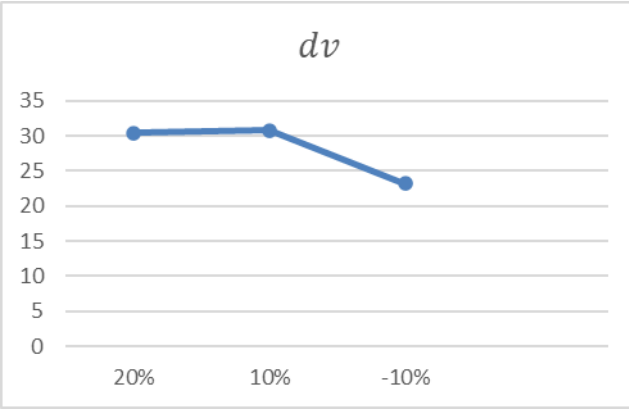


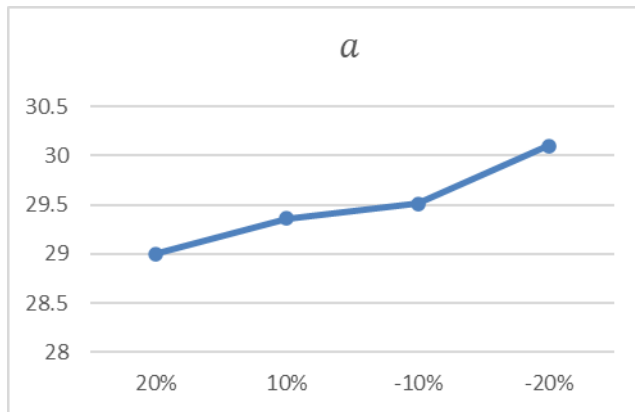
- This parameter has large values for T (around 12) and S (over 1600) with relatively low total costs (around 14) irrespective of the variations.
3. **Parameter Hcv:**
    - Shows modest variability in T (between 5.18425 and 5.81954) and S (from 0.34432 to 264.785).
    - The total cost varies from 28.38 at -20% to 32.1474 at +10%.
  4. **Parameter  $\alpha$ :**
    - The value of T ranges from 5.27261 to 6.29177, with S ranging from 98.0859 to 360.699.
    - The total cost shows a decreasing trend as  $\alpha$  decreases, going from 32.0427 at +20% to 26.6389 at -20%.
  5. **Parameter  $\phi v$ :**
    - T and S values increase as  $\phi v$  decreases.
    - Total cost decreases significantly from 30.6779 at +20% to 25.0689 at -20%.
  6. **Parameter g:**
    - There is substantial variability in T (between 5.31711 and 12.7175) and S (from 132.266 to 1664.1).
    - Total cost ranges from 14.4182 to 29.7568.
  7. **Parameter P:**
    - Shows some variability in T (between 5.3006 and 7.1743) and a wide range in SS (from 105.92 to 725.456).
    - The total cost varies from 23.9953 to 29.5947.
  8. **Parameter dv:**
    - TT ranges from 5.41158 to 7.60125, and SS from 58.3025 to 1011.67.
    - The total cost ranges from 23.2396 to 30.8028.
  9. **Parameter Rv:**
    - T increases with higher  $\phi v$ , while SS remains around 1658, and the total cost is relatively constant around 14.
  10. **Parameter Ss:**
    - T decreases with larger values (from 9.80456 to 8.09743), and S ranges from 1634.98 to 1700.1.
    - The total cost shows a decreasing trend from 16.943 to 15.0843.
  11. **Parameter a:**
    - T remains relatively stable (between 5.6345 and 5.78422), and S ranges from 255.98 to 300.09.
    - The total cost shows a slight increase from 29.0032 to 30.0987 as a decreases.

## Graphs

Following graphs show the variation of total cost with variation in different parameters







### Conclusion:

In this study, we developed a mathematical model for integrated supply chain cost management using a two-echelon approach. By conducting a sensitivity analysis, we explored the impact of various parameters on the total supply chain cost, as well as on specific performance metrics T and S. In conclusion, this integrated supply chain cost management model using a two-echelon approach provides a robust framework for understanding and optimizing supply chain performance. By identifying critical parameters and their impacts, this model aids in making informed decisions to enhance cost efficiency and overall supply chain effectiveness. Future research could extend this model to include additional echelons and explore its application in different industry contexts to further validate and refine its utility.

### References:

- Ballou, R. H. (2007). *Business Logistics/Supply Chain Management: Planning, Organizing, and Controlling the Supply Chain* (5th ed.). Pearson Prentice Hall.
- Blanchard, D. (2010). *Supply Chain Management Best Practices* (2nd ed.). Wiley.
- Cachon, G., & Terwiesch, C. (2009). *Matching Supply with Demand: An Introduction to Operations Management* (3rd ed.). McGraw-Hill.
- Chen, Z., & Ryan, J. K. (2001). The impact of rework on the lot-sizing decisions in manufacturing systems. *IIE Transactions*, 33(5), 387-398.
- Chopra, S., & Meindl, P. (2019). *Supply Chain Management: Strategy, Planning, and Operation* (7th ed.). Pearson.
- Chopra, S., Reinhardt, G., & Dada, M. (2004). The effect of lead time on the profitability of demand-driven versus forecast-driven supply chains. *Production and Operations Management*, 13(3), 220-231.
- Christopher, M. (2016). *Logistics & Supply Chain Management* (5th ed.). Pearson.
- Disney, S. M., & Towill, D. R. (2003). The effect of vendor managed inventory (VMI) dynamics on the Bullwhip Effect in supply chains. *International Journal of Production Economics*, 85(2), 199-215.
- Fisher, M. L. (1997). What is the right supply chain for your product? *Harvard Business Review*, 75(2), 105-116.
- Flynn, B. B., Huo, B., & Zhao, X. (2010). The impact of supply chain integration on performance: A contingency and configuration approach. *Journal of Operations Management*, 28(1), 58-71.
- Graves, S. C., & Willems, S. P. (2000). Optimizing strategic safety stock placement in supply chains. *Manufacturing & Service Operations Management*, 2(1), 68-83.
- Heizer, J., Render, B., & Munson, C. (2017). *Operations Management: Sustainability and Supply Chain Management* (12th ed.). Pearson.
- Hopp, W. J., & Spearman, M. L. (2008). *Factory Physics* (3rd ed.). McGraw-Hill.
- Ivanov, D., & Dolgui, A. (2020). Viability of intertwined supply networks: Extending the supply chain resilience angles towards survivability. *International Journal of Production Research*, 58(10), 2904-2915.

- Ketchen, D. J., & Hult, G. T. M. (2007). Bridging organization theory and supply chain management: The case of best value supply chains. *Journal of Operations Management*, 25(2), 573-580.
- Lambert, D. M., Cooper, M. C., & Pagh, J. D. (1998). Supply chain management: Implementation issues and research opportunities. *The International Journal of Logistics Management*, 9(2), 1-19.
- Lee, H. L., Padmanabhan, V., & Whang, S. (1997). The bullwhip effect in supply chains. *MIT Sloan Management Review*, 38(3), 93-102.
- Li, S., Ragu-Nathan, B., Ragu-Nathan, T. S., & Rao, S. S. (2006). The impact of supply chain management practices on competitive advantage and organizational performance. *Omega*, 34(2), 107-124.
- Mentzer, J. T., DeWitt, W., Keebler, J. S., Min, S., Nix, N. W., Smith, C. D., & Zacharia, Z. G. (2001). Defining supply chain management. *Journal of Business Logistics*, 22(2), 1-25.
- Nahmias, S. (1982). Perishable inventory theory: A review. *Operations Research*, 30(4), 680-708.
- Queiroz, M. M., Ivanov, D., Dolgui, A., & Fosso Wamba, S. (2021). Impacts of epidemic outbreaks on supply chains: Mapping a research agenda amid the COVID-19 pandemic through a structured literature review. *Annals of Operations Research*, 291(1), 969-999.
- Raj, R., Jayaraman, V., & Jain, V. (2020). Supply chain management for sustainable operations. *International Journal of Production Economics*, 227, 107667.
- Sari, K. (2008). On the benefits of CPFR and VMI: A comparative simulation study. *International Journal of Production Economics*, 113(2), 575-586.
- Silva, J. E., Dinis-Carvalho, J., Sousa, R. M., & Moreira, F. (2021). Cost management in supply chains: A data-driven approach. *Procedia Manufacturing*, 51, 1071-1078.
- Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (2008). *Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies* (3rd ed.). McGraw-Hill/Irwin.
- Sodhi, M. S., & Tang, C. S. (2012). Managing supply chain risk. *International Journal of Production Economics*, 139(2), 565-571.
- Swaminathan, J. M., & Tayur, S. R. (2003). Models for supply chains in e-business. *Management Science*, 49(10), 1387-1406.
- Zipkin, P. (2000). *Foundations of Inventory Management*. McGraw-Hill.